Translational Entanglement via Collisions: How Much Quantum Information is Obtainable?

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We study collisions mediated by finite-range potentials as a tool for generating translational entanglement between unbound particles or multipartite systems. The general analysis is applied to one-dimensional scattering, where resonances and the initial phase-space distribution are shown to determine the degree of postcollisional entanglement.

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The role of entanglement as the most important resource for the processing of quantum information (QI) is widely recognized [1]. In this context, there is growing interest in entangled states of continuous variables [2–7] and their resemblance to the position and momentum entangled Einstein-Podolsky-Rosen (EPR) states [8]. Continuous-variable entanglement has been analyzed using a finite-dimensional basis for systems of *bound* particles [2], or an infinite, but countable, harmonic-oscillator basis, for field quadratures [3].

Here we aim at understanding and quantifying the entanglement of the translational degrees of freedom, by studying a ubiquitous class of processes that have thus far eluded the attention of the QI community, namely, binary collisions between *unbound particles* or multipartite systems. For two systems, initially in a pure, unbound state, the post-collision (final) entanglement is measurable by the von Neumann (VN) entropy of either of the systems, $(S_j)_f = -\mathrm{tr}\,\rho_j \log \rho_j$, where ρ_j $(j = 1, 2)$ is the reduced density matrix of system j. The nonstandard calculation of $(S_j)_f$ for *continuous* variables requires the solution of the eigenvalue equation of a continuous density operator. Our comprehensive investigation of the dependence of $(S_j)_f$ on the collision parameters has revealed the following striking conclusion: entanglement is maximized near a scattering resonance, and grows with the phase-space volume of the initial (uncorrelated) two-system state, up to a limit determined by the spectral distance between resonances. These general results may advance the course of continuous-variable QI processing and its protection from decoherence, e.g., using controlled collisions of cold atoms [5,9] or slow-light polaritons [10].

Consider two systems, 1 and 2, coupled by a *finite-range* interaction. Assuming that the interaction does not affect the internal states of each system, the initial two-system *unbound* state, $|\Psi_i\rangle$, evolves, after the interaction (collision) has ceased, as [11]

$$|\Psi_i\rangle \rightarrow |\Psi_f\rangle = [U_1(t) \otimes U_2(t)](I_{\mathrm{c.m.}} \otimes \mathsf{S})|\Psi_i\rangle, \quad (1)$$

where U_j is the free-evolution propagator of system $j = 1, 2$, $I_{\mathrm{c.m.}}$ is the identity operator of the center-of-mass (c.m.) motion of the two systems, and S is the scattering matrix for their relative motion. The post-collision single-system VN entropy is then obtainable as

$$(S_1)_f = \mathrm{tr}_1(\rho_1 \log_2 \rho_1), \quad (2)$$

$$\rho_1 = \mathrm{tr}_2[(I_{\mathrm{c.m.}} \otimes \mathsf{S})|\Psi_i\rangle\langle\Psi_i|(I_{\mathrm{c.m.}} \otimes \mathsf{S})^\dagger], \quad (3)$$

where we have used the VN entropy's invariance under unitary transformations. The entanglement is a function solely of the scattering matrix and the initial wave function. Since the diagonalization of the continuous-variable reduced density matrix ρ_1, as required to compute $(S_1)_f$, is generally intractable, simplifications and approximations are imperative.

We first consider two unbound particles of equal mass, such that their relative momentum is related to their individual momenta by $\mathbf{k}_{\mathrm{rel}} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$. For three-dimensional (3D) collisions, we shall assume that each momentum state $|\mathbf{k}_i\rangle$ can scatter onto a discrete, orthonormal set of final states with momenta $\{\mathbf{k}_f\}_{j=1}^{M}$. The replacement of continuous variables by discrete values of $\mathbf{k}_f$ and $\mathbf{k}_i$ implies the use of momentum wave packets centered around these values, whose widths are small enough for the S-matrix elements $\langle \mathbf{k}_f|\mathsf{S}|\mathbf{k}_i\rangle$ to be constant throughout the wave packet; a condition that amounts to choosing appropriate initial and final momentum wave packets, as discussed below. Let us take the initial two-particle state to be an entangled superposition

$$|\Psi_i\rangle = \sum_i c_i |\mathbf{k}_i\rangle \otimes |-\mathbf{k}_i\rangle = \sum_i c_i |\mathbf{K}_{\mathrm{c.m.}} = \mathbf{0}\rangle \otimes |\mathbf{k}_{\mathrm{rel}} = \mathbf{k}_i\rangle. \quad (4)$$

Following the collision, (4) evolves, according to the superposition principle, into

$$|\Psi_f\rangle = \sum_i \sum_{\mathbf{k}_f} c_i \langle \mathbf{k}_f|\mathsf{S}|\mathbf{k}_i\rangle |-\mathbf{k}_f\rangle \otimes |\mathbf{k}_f\rangle, \quad (5)$$

the sum running over $\mathbf{k}_{\mathrm{rel}} = \mathbf{k}_f$. The resulting density matrix $|\Psi_f\rangle\langle\Psi_f|$ of the system can be traced out to yield $(\rho_1)_f = \sum_{\mathbf{k}_f} |\sum_i \langle \mathbf{k}_f|\mathsf{S}|\mathbf{k}_i\rangle|^2 |\mathbf{k}_f\rangle\langle\mathbf{k}_f|$. On the other hand, the initial entropy of the state (4) is readily seen to be $-\sum_i |c_i|^2 \log_2 |c_i|^2$. Hence, to zeroth order in the momentum widths of the initial wave packets, the change $\Delta S_1^{(0)}$ in

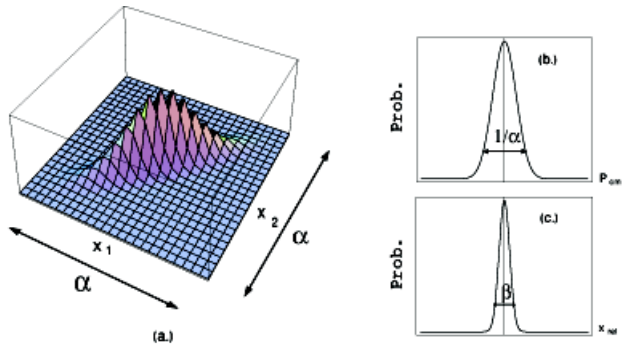

FIG. 1 (color online). (a) Phase-space distribution of 1D-entangled EPR pair with spatial width β in a box of size α. (b) Probability density as a function of c.m. momentum. (c) Same, as a function of $x_{\rm rel} = x_1 - x_2$.

the VN entropy of particle 1 as a result of the collision is

$$\Delta S_1^{(0)} = -\sum_{\mathbf{k}_f} \left| \sum_i c_i \mathsf{S}_{\mathbf{k}_f,\mathbf{k}_i} \right|^2 \log_2\left(\left| \sum_i c_i \mathsf{S}_{\mathbf{k}_f,\mathbf{k}_i} \right|^2 \right) + \sum_i |c_i|^2 \log_2 |c_i|^2, \tag{6}$$

where $\mathsf{S}_{\mathbf{k},\mathbf{k}'} \equiv \langle \mathbf{k}|\mathsf{S}|\mathbf{k}'\rangle$. In the case $c_1 = 1$, when (4) has the unentangled (product) form $|\mathbf{k}_i\rangle \otimes |-\mathbf{k}_i\rangle$, (6) reduces to $\Delta S_1^{(0)} = -\sum_{\mathbf{k}_f} |\mathsf{S}_{\mathbf{k}_f,\mathbf{k}_i}|^2 \log_2(|\mathsf{S}_{\mathbf{k}_f,\mathbf{k}_i}|^2)$. This result is in complete correspondence with the classical Boltzmann law for entropy change in collisions [12], if we identify $|\mathsf{S}_{\mathbf{k}_f,\mathbf{k}_i}|^2$ with the transition probability from an initial to a final momentum state. By contrast, the interferences of different scattering channels in (4) for two or more $c_i \neq 0$, render ΔS_1 nonclassical.

We now extend our analysis to binary collisions of two N-partite bosonic systems. The collisional QI (VN entropy) change is determined, for a given multipartite state $|\Psi_i\rangle$, by those final states that are accessible via appreciable, especially *near-resonant*, S-matrix elements. This implies that the postcollisional entanglement is effectively confined to a *finite* phase-space volume, whose size is determined by the convolution of the S-matrix spectrum with the initial distribution. We can therefore obtain an upper bound on the QI change in a collision in terms of the size of the effectively occupied phase-space volume, by deriving a quantum version of the classical sampling theorem [13]. To that end, we assume that the $2N$-partite wave function $|\Psi\rangle$ is confined to a spatial volume $V_x = L_x L_y L_z$ and to a momentum volume $V_k = \delta k_x \delta k_y \delta k_z$, determined by the *post-collision* momentum and position uncertainties. The momentum-space distribution $\langle\{\mathbf{k}_i\}_1, \{\mathbf{k}_i\}_2|\Psi\rangle$ can be expanded in the $|\{\mathbf{k}_i\}_1, \{\mathbf{k}_i\}_2\rangle \equiv |\mathbf{k}_1^{(1)}, \ldots, \mathbf{k}_N^{(1)}\rangle_1 \times |\mathbf{k}_1^{(2)}, \ldots, \mathbf{k}_N^{(2)}\rangle_2$ basis as a Fourier series whose Fourier coefficients, $a_{\{\mathbf{n}_i\},\{\mathbf{m}_i\}} \equiv a_{\mathbf{n}_1,\ldots,\mathbf{n}_N,\mathbf{m}_1,\ldots,\mathbf{m}_N}$, are related to the spatial wave function $\Psi_x(\mathbf{r}_1^{(1)}, \ldots, \mathbf{r}_N^{(1)}, \mathbf{r}_1^{(2)}, \ldots, \mathbf{r}_N^{(2)}) \equiv \langle\{\mathbf{r}_i\}_1, \{\mathbf{r}_i\}_2|\Psi\rangle$ by

$$a_{\{\mathbf{n}_i\},\{\mathbf{m}_i\}} = \left(\frac{2\pi}{V_k}\right)^{3N} \Psi_x\left(-\frac{2\pi n_{1,x}}{\delta k_x}, -\frac{2\pi n_{1,y}}{\delta k_y}, \ldots, -\frac{2\pi m_{N,z}}{\delta k_z}\right). \tag{7}$$

Since each argument $\mathbf{r}_i^j$ $(j = 1, 2, i = 1, \ldots, N)$ in $\Psi_x(\mathbf{r}_1^{(1)}, \ldots, \mathbf{r}_N^{(1)}, \mathbf{r}_1^{(2)}, \ldots, \mathbf{r}_N^{(2)})$ is confined to the finite region V_x, each $n_{i,k}$ and $m_{i,k}$ $(i = 1, \ldots, N)$ in (7) must satisfy $0 \leq -[(2\pi n_{i,j})/(\delta k_j)], -[(2\pi m_{i,j})/(\delta k_j)] \leq L_j$ for $j = 1, 2, 3$ and $i = 1, \ldots, N$, so there can be at most $(\delta k_j L_j)/(2\pi)$ nonzero n_j (and m_j) values. Thus, the continuous $2N$-partite system can be effectively described by a finite set of nonzero coefficients, $a_{\{\mathbf{n}_i\},\{\mathbf{m}_i\}}$, and the reduced state space of either of the colliding N-particle systems is specified by a set of $[(V_k V_x)/(2\pi)^3]^N$ of these coefficients. Since the maximal single-particle entropy in a d-dimensional space satisfies $S \leq \log_2 d$, an upper bound for the entanglement is then

$$S_1 = S_2 \leq N \log_2\left(\frac{V_k V_x}{(2\pi)^3}\right). \tag{8}$$

It is instructive to compare collisionally entangled particles to an EPR pair [8], consisting of two particles, prepared by dissociation (half-collision) [4,6,7], which can be approximated by a confined Gaussian wave packet for the relative motion, $\Psi(\mathbf{r}_1, \mathbf{r}_2) = (\frac{2}{\pi\alpha\beta})^{1/2} e^{-[(\mathbf{r}_1^2+\mathbf{r}_2^2)/\alpha^2]} e^{-[(\mathbf{r}_2-\mathbf{r}_1)^2/\beta^2]}$, with $\beta \ll \alpha$ (Fig. 1). One can deduce from (8) that the maximal entanglement for such a state is $\log_2(\frac{\alpha}{\beta})$, the log of the squeezing parameter s [4]. It is unclear how large is the achievable s-value for collisions of unbound particles.

To investigate this, we shall henceforth restrict ourselves to 1D collisions of two unbound particles with mass m. The effect of the S-matrix on single-particle momentum eigenkets is then $\mathsf{S}|k\rangle = T(k)|k\rangle + R(k)|-k\rangle$, where $T(k)$ and $R(k)$ are, respectively, the transmission and reflection coefficients of the interaction potential $V(x_{\rm rel})$. Using this, and taking $|\Psi_i\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, where the initial wave packets $|\psi_1\rangle, |\psi_2\rangle$ are *orthogonal*, we can simplify (3) in several steps: Inserting the appropriate identity element $I = \int dK_{\rm c.m.} dk_{\rm rel} |K_{\rm c.m.}, k_{\rm rel}\rangle\langle K_{\rm c.m.}, k_{\rm rel}|$, where $K_{\rm c.m.} = k_1 + k_2$, $k_{\rm rel} = \frac{k_1 - k_2}{2}$, and integrating over $K_{\rm c.m.}, k_{\rm rel}$, we obtain $(I_{\rm c.m.} \otimes S_{\rm rel})|k_1, k_2\rangle = T(k_{\rm rel})|K_{\rm c.m.}, k_{\rm rel}\rangle + R(k_{\rm rel}) \times |K_{\rm c.m.}, -k_{\rm rel}\rangle$. Now we can rewrite $(\rho_1)_f$ in the momentum basis, $\tilde{\rho}_{AB} = \int dk_1 dk_2 dk_1' dk_2' |k_1, k_2\rangle\langle k_1, k_2| (I_{\rm c.m.} \otimes S_{\rm rel}) \times |\psi_{12}\rangle\langle\psi_{12}| (I_{\rm c.m.} \otimes S_{\rm rel})^\dagger |k_1', k_2'\rangle\langle k_1', k_2'|$, and take the trace over particle 2, obtaining

$$\tilde{\rho}_1 = \int dk_1 dk_1' dk \left[T\left(\frac{k_1 - k}{2}\right)\psi_1(k_1)\psi_2(k) + R\left(\frac{k_1 - k}{2}\right)\psi_1(k)\psi_2(k_1) \right] \left[T^*\left(\frac{k_1' - k}{2}\right)\psi_1^*(k_1')\psi_2^*(k) + R^*\left(\frac{k_1' - k}{2}\right)\psi_1^*(k)\psi_2^*(k_1') \right]. \tag{9}$$

The cross terms R^*T and T^*R vanish owing to the assumed orthogonality of the initial wave packets in momentum space. We then have

$$\rho_1 = \rho_1^T + \rho_1^R, \tag{10}$$

$$\rho_1^T = \int dk_1 dk_1' dk |k_1\rangle \times \langle k_1'|\psi_1(k_1)\psi_1^*(k_1')|\psi_2(k)|^2 T\left(\frac{k_1 - k}{2}\right) T^*\left(\frac{k_1' - k}{2}\right), \tag{11}$$

$$\rho_1^R = \int dk_1 dk_1' dk |k_1\rangle \times \langle k_1'|\psi_2(k_1)\psi_2^*(k_1')|\psi_1(k)|^2 R\left(\frac{k_1 - k}{2}\right) R^*\left(\frac{k_1' - k}{2}\right). \tag{12}$$

Thus, the 1D density operator splits naturally into ''transmitted'' and ''reflected'' parts, which are orthogonal in the sense that $\rho_1^T \rho_1^R = 0$. As a result, the set of eigenvalues of ρ_1 is given by the union of the sets of eigenvalues of ρ_1^T and ρ_1^R.

In order to understand the near-resonance behavior of [10–12], we perform a series expansion of ρ_1^T and ρ_1^R to second order in the momentum width of the wave packets, σ. As a result, the momentum spread $\Delta k^2 \equiv \langle k - k_0\rangle^2 \sim \sigma^2$ turns the eigenvalues of (10) into

$$\Delta S_1^{(2)} = -\lambda_T^{(2)} \log_2 \lambda_T^{(2)} - \lambda_R^{(2)} \log_2 \lambda_R^{(2)} \tag{13}$$

$$\lambda_T^{(2)} = |T(k_0)|^2 + \frac{\Delta k^2}{4} \frac{d^2}{dk^2} |T(k)|^2_{k=k_0} \tag{14}$$

$$\lambda_R^{(2)} = |R(k_0)|^2 - \frac{\Delta k^2}{4} \frac{d^2}{dk^2} |T(k)|^2_{k=k_0}. \tag{15}$$

This analysis, verified by the numerical case study below, shows that initial narrow-width wave packets are expected to yield double peaks of the entanglement on both sides of a resonance, where $|T(k)|^2 = |R(k)|^2 = \frac{1}{2}$ and $T(k)$ varies strongly, with a nonzero dip at resonance. In the limit $\Delta k \to 0$, the entanglement vanishes when $|T(k)| = 1$ (at resonance) or when $|R(k)| = 1$ (both wave packets are reflected entirely).

The bound (8) may be invoked to estimate ΔS_1 for appreciable 1D momentum widths, $\sigma \gg \Gamma$, i.e., when the initial wave packet width is much larger than that of the resonance. The post-collision wave function can then be written as the sum of transmitted and reflected wave packets, using Eq. (10). Each wave packet is initially confined, in k-space, to a 1D region of dimensions $\Delta k \sim \sigma$. After the collision they are modulated by the transmission and reflection coefficients, the smallest scale of change for both given by Γ. Since the smallest scale of change of a function is the largest scale of change of its Fourier transform, we can deduce that the emerging wave packets will be confined in x-space to a box of dimensions $\Delta x_j \sim \frac{1}{\Gamma}$. Hence, applying the bound (8), we can deduce that $(S_j)_{\max} \sim \log_2(\frac{\sigma}{\Gamma})$ $(j = 1, 2)$. In fact, *the growth saturates as σ exceeds the distance between resonances* (in momentum space).

In order to corroborate the above analytical estimates, we perform a numerical case study, taking the interaction potential to be described by a 1D double delta of width a, $V(x_{\rm rel}) = V[\delta(x_{\rm rel} - a) + \delta(x_{\rm rel} + a)]$, for which the transmission and reflection coefficients are known to be

$$T(k) = \frac{4km}{(V\hbar^2 e^{2\imath ak})^2 + (4km + \imath\hbar^2 V)^2}, \tag{16}$$

$$R(k) = -\frac{\hbar^2 V[4\imath km(1 + e^{4\imath km}) + (-1 + e^{4\imath km})\hbar^2 V]}{e^{2\imath ak}[e^{4\imath km}\hbar^4 V^2 + (4km + \imath\hbar^2 V)^2]}. \tag{17}$$

The initial wave functions are taken to be two distant, counterpropagating Gaussians, $\psi_{1,2}(k) = (\frac{1}{\pi\sigma^2})^{1/4} e^{-[(k \mp k_0)^2/(2\sigma^2)] \pm \imath k x_0}$, with $x_0 < 0$, $k_0 > 0$. Note that this implies $k_{\rm rel_0} = \frac{k_{01} - k_{02}}{2} = k_0$. Finding the eigenvalues of ρ_1^T (or ρ_1^R) involves solving an integral equation of the form , where ϕ_λ are the eigenvectors and $\rho_1^{T(R)}(k, k') \equiv \langle k|\rho_1^{T(R)}|k'\rangle$ is the kernel of $\rho_1^{T(R)}$. Examining (11), we note that $\rho_1^T(k, k')$ becomes negligible for $|k - k_0| \gg \sigma$ and $|k' + k_0| \gg \sigma$. Hence, the integral can be approximated by the finite sum, $\sum_{k'=-\infty}^{\infty} \Delta k \rho_1^T(k, k')\phi_\lambda(k') = \lambda\phi_\lambda(k)$. The problem then reduces to finding the eigenvalues of a square matrix with entries $\Delta k \rho_1^{T(R)}(k, k')$, where k and k' range from $\pm k_0 - 3 * \sigma$ to $k_0 + 3 * \sigma$ in steps of δk, which must be much smaller than the scale of change of both the potential and the wave packet, so as to approximate the continuous integration faithfully (to within 99.992%). To verify the validity of the approximation, care is taken to have $|\Sigma_n \lambda_n - 1| < 0.0001$. Such a small deviation can result in an error of 0.0013 in the final calculation of the entropy. The analytical ΔS_1, given by (13), is compared in Fig. 2 with the numerical results of the simulation for different σ. The agreement is very good.

We note the most interesting features of the numerical results (Fig. 2): (i) For small initial momentum widths, such that $\sigma \ll \Gamma$ [i.e., for the last few peaks in Fig. 2 (inset)], the numerical results confirm the analytical prediction (13), whereby the entropy must have a dip at resonance, $|T|^2 = 1$. On the other hand, ΔS_1 is *maximal* at resonance for $\sigma \sim \Gamma$ or greater. (ii) We must control the spacing of resonances, Δk, and their widths, Γ, in order to attain $\Delta k \gg \sigma \gg \Gamma$, which is the optimal range for large ΔS_1. Hence, practically unlimited ΔS_1, indicating an approximate translational EPR state [6], is anticipated for near-resonant collisions, either if the potential has a *single resonance* only (Δk very large), or if the resonance width Γ is very narrow. (iii) The dependence of ΔS_1 is insensitive to fluctuations in the initial state and is thus *noise resilient*.

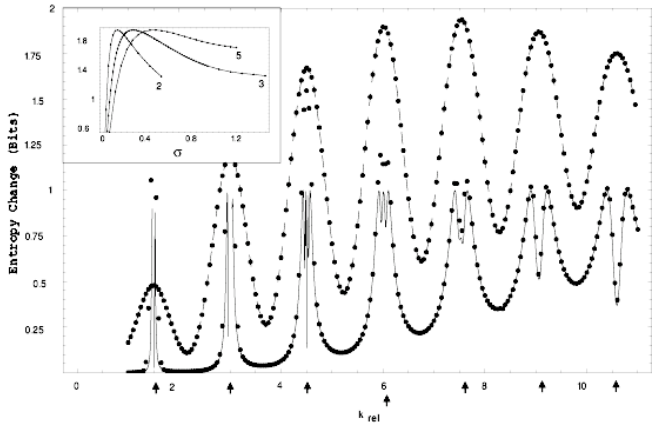

FIG. 2. Simulation results for the single-particle change in VN entropy, ΔS_1 in 1D collisions for the delta potential, as a function of the relative momentum. Lower dots: for small width $\sigma = 0.1 \ll \Gamma$, fitted to the theoretical derivation [Eq. (13)]. Upper dots: large width, $\sigma = 0.6 \sim \Gamma$. The arrows indicate the resonances of the transmission coefficient, where $|T|^2 = 1$. Inset: variation of the VN entropy as a function of the wave packet width, σ, for three relative momenta, corresponding to peaks 2, 3, and 5 of the transmission coefficient.

In conclusion, we have proposed and studied the formation of continuous-variable entanglement in controlled collisions between quasifree particles, or quantized collective excitations, interacting as fictitious particles, e.g., impurity-atom collisions with Bose-Einstein condenstates (BECs) [14], or collisions of slow-light polaritons in gases or solids [10]. Cold atoms [5] or slow-light polaritons [10], free to move in 1D, but confined in the remaining 2D by an optical lattice or a waveguide, as well as small-angle collisions of fast-particles [4,11], are suitable candidates. Appropriate ensemble averaging can be used to apply these results to the description of the transition of large quantum systems to entropic equilibrium and classicality via binary collisions. The maximal amount of entanglement entropy has been shown to scale logarithmically with the position-momentum uncertainty product (phase-space volume) of the colliding wave packets, only for *large* wave packet widths compared to the resonances' width, but less than the distance between resonances. Translational entanglement of unbound particles has been shown to be highly noise resilient in certain cases. This reflects an essential aspect of such processes: ''most'' of their continuous degrees of freedom may be redundant, and hence effectively protect their entanglement against spurious fluctuations. These results provide, for the first time, guidelines for the possible use of unbound particles in QI processing of continuous variables, as well as conditions for the suppression of decoherence by minimization of the collision-induced entanglement.

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