MRI Primer, Exercise #9 : Solution

Question 1.

a. Doubling the gradient corresponds to doubling both Δk and k_{max}, so

$$FOV \rightarrow FOV/2$$
$$\Delta x \rightarrow \Delta x/2$$

Since $SNR \propto M_0 \Delta x \sqrt{T}$, we have

$$SNR \rightarrow SNR/2$$

b. Doubling T and N (while keeping the dwell time and G constant) means

$$k_{max} \rightarrow 2 \times k_{max}$$
$$\Delta k \rightarrow \Delta k$$

so

$$FOV \rightarrow FOV$$
$$\Delta x \rightarrow \Delta x/2$$

Thus

$$SNR \rightarrow SNR/\sqrt{2}$$

c. Keeping the voxel size fixed corresponds to keeping the same k_{max}. To double the FOV you would then have to take

$$\Delta k \rightarrow \Delta k/2$$

So:

What we need is

$$k_{max} \rightarrow k_{max}$$
$$\Delta k \rightarrow \Delta k/2$$

This can be achieved in an infinite number of ways. Indeed, take

$$G \rightarrow \alpha \times G$$
$$T \rightarrow \tfrac{1}{\alpha} \times T$$

This would result in

$$k_{max} = \gamma GT \rightarrow \alpha \times \tfrac{1}{\alpha} \times \gamma GT = \gamma GT \quad \text{(no change)}$$
$$\Delta k = \gamma G \delta t \rightarrow \gamma \alpha G \delta t^{(old)} \stackrel{!}{=} \tfrac{1}{2} \gamma G \delta t^{(new)} \Rightarrow \qquad \delta t^{(new)} = 2\alpha \delta t^{(old)}$$

The SNR would change as

$$SNR \rightarrow \sqrt{T^{(new)}} SNR = \frac{SNR}{\sqrt{\alpha}}$$

depending on your choice of α. Thus, **any** answer is possible! The SNR can increase (by taking longer times) or decrease (by taking shorter times), or even stay the same (same G and T, but halving the dwell time).

Question 2.
No. The signal would increase by a factor of $\sqrt{2}$, but so would the noise when adding both measurements. The SNR will not change.

Question 3.
One way would be to use (steady-state, spoiled) spin echo imaging and to acquire two images, one with TR much larger than any T_1, another with TR~T_1. The SE contrast is given by:

$$I(\mathbf{r}) \propto M_0(\mathbf{r})\left(1 - e^{-TR/T_1(\mathbf{r})}\right) e^{-TE/T_2(\mathbf{r})}$$

Thus the two images obtained will have intensities (as a function of position) proportional to:

$$I_1(\mathbf{r}) \propto M_0(\mathbf{r}) e^{-TE/T_2(\mathbf{r})}$$
$$I_2(\mathbf{r}) \propto M_0(\mathbf{r})\left(1 - e^{-TR/T_1(\mathbf{r})}\right) e^{-TE/T_2(\mathbf{r})}$$

Dividing yields

$$f(\mathbf{r}) = \frac{I_2}{I_1} = 1 - e^{-TR/T_1(\mathbf{r})}$$

Since f(**r**) & TR are known, it is simple to get a T_1 map. In practice however this is not often done, due to SNR limitations: in regions where noise is significant, the ratio I_2/I_1 is very inaccurate. A better way might be to use an inversion-recovery sequence of the type discussed in question 4 (although there it's used for water/fat suppression) to acquire multiple T_1-weighted images, and then to fit – per voxel – the intensity to $1\text{-}2e^{-\tau/T1}$.

Question 4

a.

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \\ -M_0 \end{pmatrix}$$

b. After a time τ, using ex. 3's results (with M_a=0, M_b=-M_0):

$$\mathbf{M}(\tau) = \begin{pmatrix} 0 \\ 0 \\ M_0\left[1-2e^{-\tau/T_1}\right] \end{pmatrix}$$

c.

$$M_{xy} = M_0\left[1-2e^{-\tau/T_1}\right]$$

d. To make the signal vanish, choose τ such that $1-2e^{-\tau/T_1}$ vanishes:

$$\tau = \ln(2)T_1$$

Since different tissues have different T_1 values, this can be used to null one signal's tissue while leaving the other's signal intact. For T_1~1 sec,

$$\tau \approx 0.69 \text{ sec}$$