

MRI Primer, Exercise #9 : Solution

**Question 1.**

a. Doubling the gradient corresponds to doubling both  $\Delta k$  and  $k_{\max}$ , so

$$\begin{aligned} \text{FOV} &\rightarrow \text{FOV} / 2 \\ \Delta x &\rightarrow \Delta x / 2 \end{aligned}$$

Since  $\text{SNR} \propto M_0 \Delta x \sqrt{T}$ , we have

$$\text{SNR} \rightarrow \text{SNR} / 2$$

b. Doubling T and N (while keeping the dwell time and G constant) means

$$\begin{aligned} k_{\max} &\rightarrow 2 \times k_{\max} \\ \Delta k &\rightarrow \Delta k \end{aligned}$$

so

$$\begin{aligned} \text{FOV} &\rightarrow \text{FOV} \\ \Delta x &\rightarrow \Delta x / 2 \end{aligned}$$

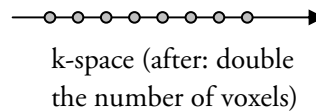
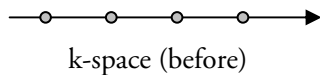
Thus

$$\text{SNR} \rightarrow \text{SNR} / \sqrt{2}$$

c. Keeping the voxel size fixed corresponds to keeping the same  $k_{\max}$ . To double the FOV you would then have to take

$$\Delta k \rightarrow \Delta k / 2$$

So:



What we need is

$$\begin{aligned} k_{\max} &\rightarrow k_{\max} \\ \Delta k &\rightarrow \Delta k / 2 \end{aligned}$$

This can be achieved in an infinite number of ways. Indeed, take

$$G \rightarrow \alpha \times G$$

$$T \rightarrow \frac{1}{\alpha} \times T$$

This would result in

$$k_{\max} = \gamma GT \rightarrow \alpha \times \frac{1}{\alpha} \times \gamma GT = \gamma GT \quad (\text{no change})$$

$$\Delta k = \gamma G \delta t \rightarrow \gamma \alpha G \delta t^{(\text{old})} = \frac{1}{2} \gamma G \delta t^{(\text{new})} \Rightarrow \delta t^{(\text{new})} = 2\alpha \delta t^{(\text{old})}$$

The SNR would change as

$$\text{SNR} \rightarrow \sqrt{T^{(\text{new})}} \text{SNR} = \frac{\text{SNR}}{\sqrt{\alpha}}$$

depending on your choice of  $\alpha$ . Thus, **any** answer is possible! The SNR can increase (by taking longer times) or decrease (by taking shorter times), or even stay the same (same  $G$  and  $T$ , but halving the dwell time).

### Question 2.

No. The signal would increase by a factor of  $\sqrt{2}$ , but so would the noise when adding both measurements. The SNR will not change.

### Question 3.

One way would be to use (steady-state, spoiled) spin echo imaging and to acquire two images, one with  $TR$  much larger than any  $T_1$ , another with  $TR \sim T_1$ . The SE contrast is given by:

$$I(\mathbf{r}) \propto M_0(\mathbf{r}) \left(1 - e^{-TR/T_1(\mathbf{r})}\right) e^{-TE/T_2(\mathbf{r})}$$

Thus the two images obtained will have intensities (as a function of position) proportional to:

$$I_1(\mathbf{r}) \propto M_0(\mathbf{r}) e^{-TE/T_2(\mathbf{r})}$$

$$I_2(\mathbf{r}) \propto M_0(\mathbf{r}) \left(1 - e^{-TR/T_1(\mathbf{r})}\right) e^{-TE/T_2(\mathbf{r})}$$

Dividing yields

$$f(\mathbf{r}) = \frac{I_2}{I_1} = 1 - e^{-TR/T_1(\mathbf{r})}$$

Since  $f(\mathbf{r})$  &  $TR$  are known, it is simple to get a  $T_1$  map. In practice however this is not often done, due to SNR limitations: in regions where noise is significant, the ratio  $I_2/I_1$  is very inaccurate. A better way might be to use an inversion-recovery sequence of the type discussed in question 4 (although there it's used for water/fat suppression) to acquire multiple  $T_1$ -weighted images, and then to fit – per voxel – the intensity to  $1 - 2e^{-\tau/T_1}$ .

#### Question 4

a.

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \\ -M_0 \end{pmatrix}$$

b. After a time  $\tau$ , using ex. 3's results (with  $M_a=0$ ,  $M_b=-M_0$ ):

$$\mathbf{M}(\tau) = \begin{pmatrix} 0 \\ 0 \\ M_0 [1 - 2e^{-\tau/T_1}] \end{pmatrix}$$

c.

$$M_{xy} = M_0 [1 - 2e^{-\tau/T_1}]$$

d. To make the signal vanish, choose  $\tau$  such that  $1 - 2e^{-\tau/T_1}$  vanishes:

$$\tau = \ln(2)T_1$$

Since different tissues have different  $T_1$  values, this can be used to null one signal's tissue while leaving the other's signal intact. For  $T_1 \sim 1$  sec,

$$\tau \approx 0.69 \text{ sec}$$