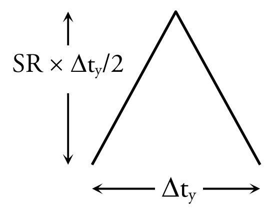
MRI Primer, Exercise #6: Solution

Question 1

Denote by SR the slew-rate. Remember that Δk is merely the area under the gradient. In our case, the gradient is triangular, so the area under it is simply the area under a triangle: 0.5 × base × height. The "base" here is Δt. The height is the value of the gradient midway through the pulse. Because the gradient is linearly increasing, this value = SR × ($\Delta t_y/2$). Combining it all together (don't forget γ!) yield:

$$\Delta k = \gamma \times 0.5 \times \Delta t_y \times (SR \times \Delta t_y/2) = 0.25 \times SR \times \Delta t_y^2$$



It is given that your field of view along the y-axis is FOV_y = 30cm = 0.3m. Hence, according to the resolution constraints, $\Delta k_y = FOV_y^{-1} = 3.33 m^{-1}$. Using the results of the previous question, $\Delta k_y = \frac{\gamma}{4} SR \Delta t_y^2$, we obtain (for SR = 200 Tesla/meter/seconds): $\Delta t_y \approx 40\ \mu\text{sec}$.

The average gradient is given by averaging over the gradient waveform:

$$\langle G(t) \rangle = \frac{1}{\Delta t_y} \int_0^{\Delta t_y} G(t')dt' = \frac{1}{\gamma \Delta t_y} \times \gamma \int_0^{\Delta t_y} G(t')dt' = \frac{\Delta k}{\gamma \Delta t_y} = \frac{SR\Delta t_y}{4} \approx 2\ \frac{\text{milliTesla}}{\text{meter}}$$

Question 2

Let's write down what we have:

$$\begin{aligned} &\Delta k_x = \Delta k_y = \frac{1}{0.3\text{m}}, \\ &N_x = N_y = \text{number of voxels}, \\ &T_x = 0.5\text{ms} \\ &\Delta t_y = 0.1\text{ms} \\ &T_{total} = T_2 = 50\text{ms} \end{aligned}$$

Computing N is straightforward now, since we know that $N_y \times (T_x + \Delta t_y) = T_{total}$, from which

$$N_x = N_y \approx 83$$

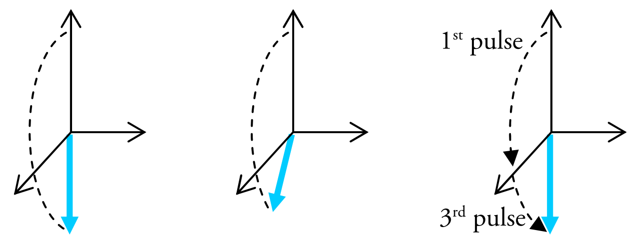
The x & y gradients are found using $\Delta k_x = \gamma G_x \Delta t_x$, $\Delta k_y = \gamma G_y \Delta t_y$, from which:

$$G_x = \frac{\Delta k_x}{\gamma \Delta t_x} = \frac{\Delta k_x}{\gamma \left(T_x / N_x\right)} = \frac{3.33\text{m}^{-1}}{\left(42\times 10^6 \frac{\text{Hz}}{\text{Tesla}}\right)\times\left(0.5\text{ms}/83\right)} \approx 13 \frac{\text{milliTesla}}{\text{meter}}$$

$$G_y = \frac{\Delta k_y}{\gamma \Delta t_y} = \frac{\Delta k_x}{\gamma \Delta t_y} = \left(\frac{\Delta t_x}{\Delta t_y}\right) G_x = \left(\frac{T_x}{N \Delta t_y}\right) G_x = \left(\frac{5}{83}\right) G_x = \frac{G_x}{16.6} \approx 0.78 \frac{\text{milliTesla}}{\text{meter}}$$

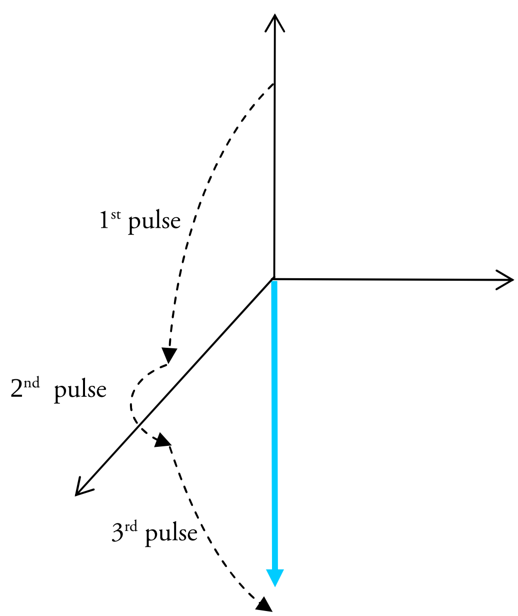
<u>Question 3</u>

This is an "answer by drawing" sort of question, so let's draw!



(a.) (b.) (c.)

The π pulse in (c.) has no effect because the axis of rotation and the spin's direction are colinear. To appreciate the composite pulse's action, I'm going to enlarge the drawing and exaggerate the angles somewhat:



(d.) Note how the middle pulse, which is almost a π pulse, compensates for the missing angle.