1. **A Riddle.** Someone gives you a sample tube containing two compartments. All you know is that one of them contains water and the other contains fat; you just don’t know which is which:

All you have is a 3T MRI machine with a one dimensional gradient. Devise **two different** experiments that will enable you tell which is which. No breaking the tube, using hydrostatics, praying for a miracle, summoning a genie, etc ... allowed. Here are a few facts:
- Water resonates at 0 Hz, fat at about 450 Hz at 3T.
- Water has a longer T2 than fat. For concreteness, let’s say that the T2 of water is 100 ms and the T2 of fat is 10 ms.

Be specific: draw pulse sequences used and explain why you’ve used them. Make up new sequences if needed. You can use more than one scan (that is, repeat the experiment with different parameters to help you tell the two apart).

Can you do this in a **single** scan (that is, in “one-shot”, without repeating the experiment)? Sketch the pulse sequence and explain your logic.

2. **Water Suppression.** It has been remarked in class that water is a major hurdle in MRS, so it’s important to find ways of suppressing its signal. Let’s work through one particular scheme, called a **binomial pulse of order 1** (sometimes denoted \(1\bar{T}\)). Instead of merely exciting the spins with, say, a 90\(_\gamma\) pulse, one uses 2 90-pulses space a time \(\tau\) apart:

   - Assuming water is on resonance, so (in a 3T machine) fat has a chemical shift of 450 Hz, explain how you would set \(\tau\) to suppress the water signal.
   - Why do you think this sequence isn’t so successful in practice?
3. Alanine (Ala for short) is an observable metabolite in MRS, present at about 0.1-1.5 mM in the human brain. It is elevated in certain CNS tumors (meningiomas) and in ischemia (restriction of blood supply). Here it is:

\[
\begin{align*}
&\text{+NH}_3^- \\
&\text{OO}^\text{1}C\text{-}2\text{C}\text{-}3\text{CH}_3 \\
&\text{H}
\end{align*}
\]

You wish to sketch its spectrum to try and predict what you’d get in a real experiment. Since all these things are tabulated, you open a book – say, de Graaf’s book, “In Vivo NMR Spectroscopy” – and look it up:

**Table 2.1** Chemical shifts, multiplicities, connectivities and scalar coupling constants\(^\star\) for \(^1\text{H}\)-containing cerebral metabolites

<table>
<thead>
<tr>
<th>Compound</th>
<th>Chemical shift (ppm)</th>
<th>Multiplicity(^\text{a})</th>
<th>Interaction</th>
<th>Scalar coupling (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alanine</td>
<td>(^2\text{CH}) 3.775</td>
<td>q</td>
<td>2-3</td>
<td>7.23</td>
</tr>
<tr>
<td></td>
<td>(^3\text{CH}_3) 1.467</td>
<td>d</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here d means doublet (one splitting due to J-coupling), and q means quartet. The quartet arises because the methine (CH) group’s proton (H) gets split by each of the protons on the methyl (CH\(_3\)) group (i.e., split thrice); the splitting is reciprocal, in the sense that the (equivalent) protons on the methyl group get split by the proton on the methyl group. The same splitting constant, J=7.23 Hz, applies both ways.

Sketch the spectrum of the Alanine metabolite at 1.5 Tesla and at 3 Tesla, using a frequency (Hertz) axis. Assume the peaks are all infinitely narrow, and that the on-resonance frequency is chosen to match that of the (unsplit) methyl group in both cases.