# MRI Primer, Exercise #2

Due 8/Dec/2009

*Important announcement*: this exercise's deadline has been extended by a week. Question #4 has been slightly changed (you'll need to redo it) and an extra question (#5) has been added. Questions #1-#3 are precisely the same as in the previous version of this exercise.

1. **Bulk Magnetization Computations**. Compute the bulk magnetization at thermal equilibrium of:
   a. A cup of water placed in the Earth's magnetic field at room temperature.
   b. A cubed millimeter of water (which corresponds to a typical voxel size in MRI) in a 3T magnetic field (which is a typical field for MRI machines) at standard body temperature (37 deg. Celsius).

   Before doing the actual computation, try to guess which will be greater (you don't have to write anything, just guess for yourself).

2. **Smallness of Nuclear Polarization.** This problem will make it clear just how weak the nuclear polarization is. Suppose you're given 1,000,000 Hydrogen atoms at room temperature, in a 3 Tesla field. At thermal equilibrium, on average, how many of the nuclear spins will point up and how many will point down? Guidance (you may ignore if you wish to solve this some other way):
   a. What is the maximal polarization of the sample, when all spins point up? E.g., for five spins: ↑↑↑↑↑.
   b. What is the actual polarization at thermal equilibrium, which you can compute? E.g., that of, say, ↑↓↑↓↑.
   c. Divide the number obtained in (a) by that obtained in (b). This should give you the percentage of ***excess*** spins pointing up (all other spins are tied up in ↑↓ pairs). Use that number to complete the question.

3. **The Effect of RF Pulses**. From this problem onwards assume you are in the rotating frame, and that you're dealing with Hydrogen nuclear spins. A spin system is at thermal equilibrium with some equilibrium magnetization $\mathbf{M} = M_0\hat{\mathbf{z}}$.
   a. Assume the spins are "on-resonance": that is, they have no offset in the rotating frame. An RF pulse is applied along the –y axis for 10 milliseconds. What should the strength of the RF field be (in Gauss) for it to tilt the spins onto the x axis? Remember: the spins precess about the RF field according to the *left hand* rule. Draw the effective field and indicate motion of the spins.
   b. Assume next the spins are "off-resonance", meaning they have an offset. Take that offset to be 25 Hertz. The same field as in part (a) is applied. How long will it have to be applied to tilt the spins onto the xy plane?
   c. Let's stick with the field strength from part (a). Explain why a system with an offset greater than 25 Hertz will not be tipped completely onto the xy plane *ever*, i.e., regardless of how long the RF field gets applied. Hint: a simple drawing will suffice. The drawings you've made in parts a & b should have helped clarify the idea.

4. **A Riddle**. A $90_{-y}$ pulse is equivalent to a $180_x$ pulse, followed by a $90_y$ pulse: true or false? Assume the spins are "on resonance"; in other words, that in the rotating frame they have no offset. (NOTE: I've had a typo in the first version of ex. #2. I've used $180_y$ instead of $180_x$. Please disregard it and redo this problem)

5. **Why is $T_2 \le T_1$?** The transverse relaxation constant, $T_2$, is always smaller than the longitudinal one, $T_1$. Here are some experimental measurements:

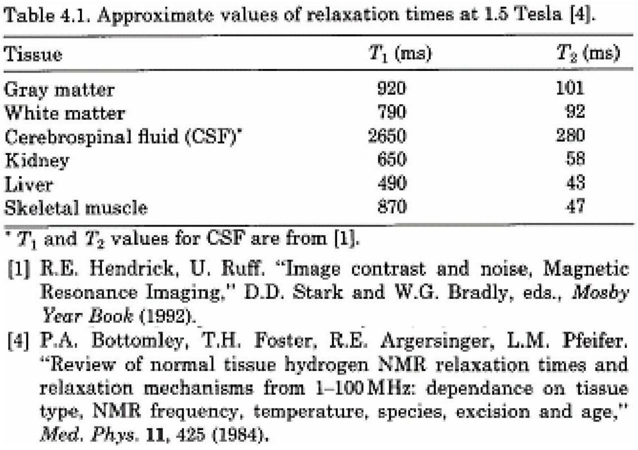

Table 4.1. Approximate values of relaxation times at 1.5 Tesla [4].

| Tissue | $T_1$ (ms) | $T_2$ (ms) |
|---|---|---|
| Gray matter | 920 | 101 |
| White matter | 790 | 92 |
| Cerebrospinal fluid (CSF)* | 2650 | 280 |
| Kidney | 650 | 58 |
| Liver | 490 | 43 |
| Skeletal muscle | 870 | 47 |

* $T_1$ and $T_2$ values for CSF are from [1].

[1] R.E. Hendrick, U. Ruff. "Image contrast and noise, Magnetic Resonance Imaging," D.D. Stark and W.G. Bradly, eds., *Mosby Year Book* (1992).

[4] P.A. Bottomley, T.H. Foster, R.E. Argersinger, L.M. Pfeifer. "Review of normal tissue hydrogen NMR relaxation times and relaxation mechanisms from 1–100 MHz: dependance on tissue type, NMR frequency, temperature, species, excision and age," *Med. Phys.* **11**, 425 (1984).

In this question we'll try to present an argument that shows that, if $T_2$ were larger than $T_1$, we would be able to create an indefinitely large magnetization vector, which is of course impossible physically. Let's assume – to make things simple and easy to visualize – that $T_2$ is *much* larger than $T_1$; for example, let's take $T_1 = 1$ sec, $T_2 = 1$ hour (for your general information, $T_2$ = hour is unheard of in MRI, and just used for illustrative purposes).

- **a.** Starting from thermal equilibrium, $\mathbf{M} = M_0\hat{\mathbf{z}}$. describe the state of the spin after a $90_{-y}$ pulse (draw a picture). Compute the magnitude of the spin vector.
- **b.** How would the spin look like (approximately) after, say, one minute? Draw a picture. Compute its magnitude.
- **c.** Assume another RF pulse is now used to tilt the magnetization (at the end of part b) to the xy plane. Draw the state of the system now. What is the magnitude of the spin vector?

Thus, we can enlarge the equilibrium magnetization. This process can be repeated indefinitely, building up the magnetization vector without limit, which is impossible. Since the only assumption we've made is that $T_2 > T_1$, it must be the problem.