MRI Primer Exercise 1

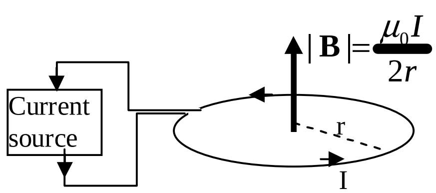
Due 24/11/2009

1. In the institute, the only human MRI machine, the Siemens' Trio, is housed next to the neurobiology building. The highest field NMR spectrometer is the DRX800 (housed right in front of Ziskind). Complete the following table, intended to give you a rough feel for the orders of magnitude involved in typical magnetic resonance experiments:

	Siemens MRI Trio human MRI machine	DRX800 NMR spectrometer	Earth's magnetic field
Precession frequency of proton ^{1}H nuclear magnetic moment		800 MHz	
Precession frequency of Phosphorous ^{31}P nuclear magnetic moment			
Precession frequency of Phosphorous ^{13}C nuclear magnetic moment			
Precession frequency of electron's magnetic moment			
Field strength (in Tesla)	3		
Field strength (in Gauss)			

To what note (on the piano) does the proton's precession frequency in the Earth's magnetic field approximately correspond? Use external resources (eg introductory science books, Wikipedia) to determine this and other quantities of interest (such as the Earth's magnetic field, the electron's gyromagnetic ratio and so forth).

2. In this problem we'll see just how hard it is to produce the enormous magnetic fields used in NMR. The easiest way to produce a magnetic field is by establishing a current, I, through a loop of wire. Using Ampere's law, which you should've encountered in an electricity & magnetism course, one can compute the magnetic field at the center of the loop to be:



where I is the current (in amperes), r is the wire's radius (in meters), and $\mu_0 = 4\pi\times10^{-7}$ (in Tesla × meters/Amperes).

   a. Suppose you took a copper wire, made a loop out of it (of radius 0.5 meters, typical of human MRI machines) and plugged it into the power supply at your home, which is capable of delivering a max. of 16 Amperes before blowing a fuse. What would be the field at the center of the loop? What would the current I_{HF} needed to create a 3 Tesla high field?

   b. A second problem which arises with copper wires is their resistance, which leads to heating. Suppose you had a current source powerful enough to generate the current I_{HF} you've found in part (a). Compute the power dissipated in the coil in Joules per second. Assume you've used a regular power cord in part (a) (i.e. the kind you use in regular household appliances). Assume it's made out of copper, and recall: (i) copper has a resistivity of $\rho = 1.7\times10^{-8}$ Ohms·meters, (ii.) the total resistance of a wire with cross section A and length l is given by $R = \rho l/A$, and (iii.) the power dissipated in a resistor R with current I flowing through it is $P=I^2R$.

c. Calculate the number of degrees a liter of water would heat up by, if it were given the thermal energy dissipated during one minute in the coil in part *(b).*

Fortunately, the low fields and the high power dissipation can both be circumvented by using super-conductive magnets. These allow us to use very high currents in very thin wires (since there is little to no power dissipation in superconductors). Moreover, they use multiple turns of the wire to increase the field considerably. That is how the large main fields in modern MRI magnets are achieved.

3. The MRI signal mostly emanates from the H_2O water molecules in our body. Estimate how many water molecules in a small mm^3 voxel in our body. Since we are made primarily out of water, this should approximate the number of water molecules in a cubic millimeter in our body fairly well, within an order of magnitude. (Hint: the number of water molecules can be estimated in several ways. For example, how much does a liter of water weigh? How much does a mm3 of water weigh? How much does a water molecule weigh? Dividing should give you a good estimate).